Analytical and numerical analyses of an unconfined aquifer test considering unsaturated zone characteristics

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[1] A 7-d, constant rate aquifer test conducted by University of Waterloo researchers at Canadian Forces Base Borden in Ontario, Canada, is useful for advancing understanding of fluid flow processes in response to pumping from an unconfined aquifer. Measured data include not only drawdown in the saturated zone but also volumetric soil moisture measured at various times and distances from the pumped well. Analytical analyses were conducted with the model published in 2001 by Moench and colleagues, which allows for gradual drainage but does not include unsaturated zone characteristics, and the model published in 2006 by Mathias and Butler, which assumes that moisture retention and relative hydraulic conductivity (RHC) in the unsaturated zone are exponential functions of pressure head. Parameters estimated with either model yield good matches between measured and simulated drawdowns in piezometers. Numerical analyses were conducted with two versions of VS2DT: one that uses traditional Brooks and Corey functional relations and one that uses a RHC function introduced in 2001 by Assouline that includes an additional parameter that accounts for soil structure and texture. The analytical model of Mathias and Butler and numerical model of VS2DT with the Assouline model both show that the RHC function must contain a fitting parameter that is different from that used in the moisture retention function. Results show the influence of field-scale heterogeneity and suggest that the RHC at the Borden site declines more rapidly with elevation above the top of the capillary fringe than would be expected if the parameters were to reflect local- or core-scale soil structure and texture.

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1. Introduction

[2] Recent literature on the interpretation of unconfined aquifer tests has demonstrated the importance of flow processes in the unsaturated zone. For at least three decades it had been assumed with some success that the influence of the unsaturated zone on the response to pumping from a well could be neglected and would have little consequence for the estimated hydraulic properties of the aquifer. The basis for this assumption lay in the pioneering theoretical work of *Boulton* [1954, 1963], *Dagan* [1967], *Neuman* [1972, 1974], and *Kroszynski and Dagan* [1975]. These and other papers relevant to unconfined aquifer tests are briefly reviewed by *Moench* [2004]. The standard model used for decades in the groundwater industry is that of *Neuman* [1974].

[3] That the influence of the unsaturated zone could be neglected in analyses of unconfined aquifer tests was an assumption that went unchallenged until papers by *Nwankwor et al.* [1984, 1992] were published pertaining to tests conducted at the Canadian Forces Base (CFB) Borden, Ontario (referred to as the Borden site in this paper). On the basis of attempts to estimate specific yield by type curve analyses and a volume balance method, *Nwankwor et al.* [1984] concluded that a model that includes effects of delayed drainage from above the water table would probably yield improved estimates of specific yield. *Nwankwor et al.* [1992] provided direct evidence that delayed drainage had occurred in the course of the test through the use of tensiometer and soil moisture measurements. In a numerical study *Narasimhan and Zhu* [1993] demonstrated the importance of including effects of drainage from the unsaturated zone in models of flow to a well in unconfined aquifers. The analysis of an aquifer test conducted at Cape Cod, Massachusetts, carried out by *Moench et al.* [2001] supports the work of *Nwankwor et al.* [1984], *Nwankwor et al.* [1992], and *Narasimhan and Zhu* [1993]. By means of a numerical model *Akindunni and Gillham* [1992] used exponential functional relations to characterize the hydraulic properties of the unsaturated zone and were able to explain the trends found in the aquifer test data of *Nwankwor et al.* [1992]. In another numerical study *El-Kadi* [2005] examined but only partially verified the validity of the generalized Richards equation for unconfined aquifer test analysis.

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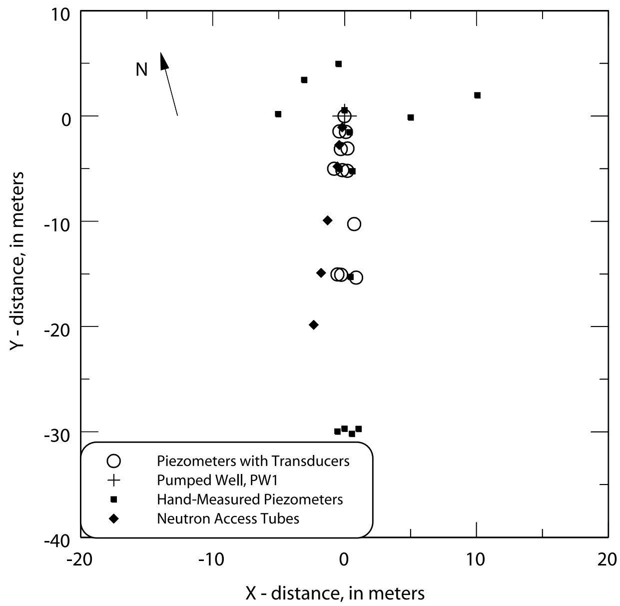

Figure 1. Plan view of the Borden test site showing the positions of the pumped well, the piezometers with transducers, the piezometers measured by hand, and the neutron access tubes.

[4] In an attempt to evaluate unsaturated zone characteristics from aquifer test data at the Cape Cod, Massachusetts, site, *Moench* [2003] used an analytical model and the USGS numerical model VS2DT [*Lappala et al.*, 1987; *Healy*, 1990] using *Brooks and Corey* [1964] functional relations. It was found that the estimated soil moisture retention looked like that which would be expected to be seen for fine-grained materials, not like that of the coarse-grained materials of which the aquifer was actually composed. In the absence of corroborating soil moisture data and accepting the validity of the Brooks and Corey model, the result was explained as being due to the existence of interbedded fine-grained material, heterogeneity, and the large scale of the aquifer test.

[5] In response to a recognition of the likely importance of unsaturated zone characteristics in analyses of unconfined aquifer tests, two new analytical solutions for flow to a well in an unconfined aquifer have recently become available [*Mathias and Butler*, 2006; *Tartakovsky and Neuman*, 2007]. Both of these models couple groundwater flow in the saturated zone with flow in the unsaturated zone and follow the original work of *Kroszynski and Dagan* [1975] that is based on a linearized Richards equation. These three models assume that volumetric soil moisture and relative hydraulic conductivity (RHC) are exponential functions of pressure head above the top of the capillary fringe.

[6] *Bevan* [2002] and *Bevan et al.* [2005] describe a detailed 7-d aquifer test conducted in August 2001 at the Borden site that is, in this author's opinion, a benchmark test. Analysis of this test is the subject of this paper. The test is unique in that volumetric soil moisture measurements were made at various times and locations simultaneous with the monitoring of drawdown in the saturated zone. By comparing the soil moisture profiles with estimates of the depth of the water table, *Bevan* [2002] concluded that the capillary fringe became elongated in the course of the test. A similar finding was reported for another test at the Borden site by *Nwankwor et al.* [1992]. The subject is of interest to groundwater hydrologists as it relates to the interface between the saturated and unsaturated zones [see *Berkowitz et al.*, 2004].

[7] In this paper, using analytical and numerical models, drawdown data from the August 2001 aquifer test at the Borden site are used to quantify the hydraulic properties of both the saturated zone and the unsaturated zone at the site. The motivation for the paper is twofold: (1) to further investigate the question, originally posed by *Moench* [2003], of whether drawdown data from an aquifer test can be used to estimate large-scale soil moisture characteristics, and (2) to investigate the validity of alternative analytical and numerical models for analysis of unconfined aquifer tests.

2. Description of the Aquifer Test

[8] The hydrogeology of the area in the vicinity of the aquifer test has been described by *MacFarlane et al.* [1983]. *Bevan* [2002] and *Bevan et al.* [2005] describe the salient details pertaining to the design and operation of the August 2001 test.

[9] *Bevan et al.* [2005] describe the aquifer at the site of the pumping test as composed mostly of unconsolidated medium-grained sand that is of glaciodeltaic and glaciofluvial origin and overlies a clayey silt aquitard at a depth of about 9 m below land surface. It can be locally heterogeneous because of discontinuous beds of fine-, medium-, and coarse-grained sand. Relevant to this paper as it may relate to soil structure and texture in the unsaturated zone is the detailed description by *Sudicky* [1986] of 32 cores of aquifer material from the Borden site. The cores were spaced at 1 m intervals along two core lines (20 and 13 m in length) perpendicular to one another and that cross at their centers. The 2-m-long cores were taken from depths of between 2.5 and 4.5 m below ground surface. Upon examination of the cores *Sudicky* [1986, p. 2073] found

> . . .numerous lenses of coarse to silty fine-grained sand embedded in a fine- to medium-grained sand. The contact between zones having a large textural contrast was usually sharp and near-horizontal across the cores. The thickness of individual beds generally varied from a few centimeters to a few tens of centimeters, with the material within each bed being relatively homogeneous in texture, although fine laminations on the order of a millimeter to a few millimeters thickness were sometimes encountered.

[10] Figure 1 is a plan view of the test site showing the location of the pumped well (PW1), 11 piezometers with transducers, 12 of the 14 piezometers that were measured by hand, and 6 neutron access tubes. Figure 2 is a vertical section showing the land surface, the position of the initial water table, the base of the aquifer, the locations of the transducer-measured and hand-measured piezometers, the location of the pumped well screen, and the location of the neutron access tubes. Radial distance and depth locations of the piezometers and of the neutron access tubes are plotted using data provided by *Bevan* [2002] and M. J. Bevan (written communication, March 2003). *Nwankwor et al.* [1984] indicate that the volumetric water content at full saturation (i.e., porosity) is 0.37 at the site of the aquifer test.

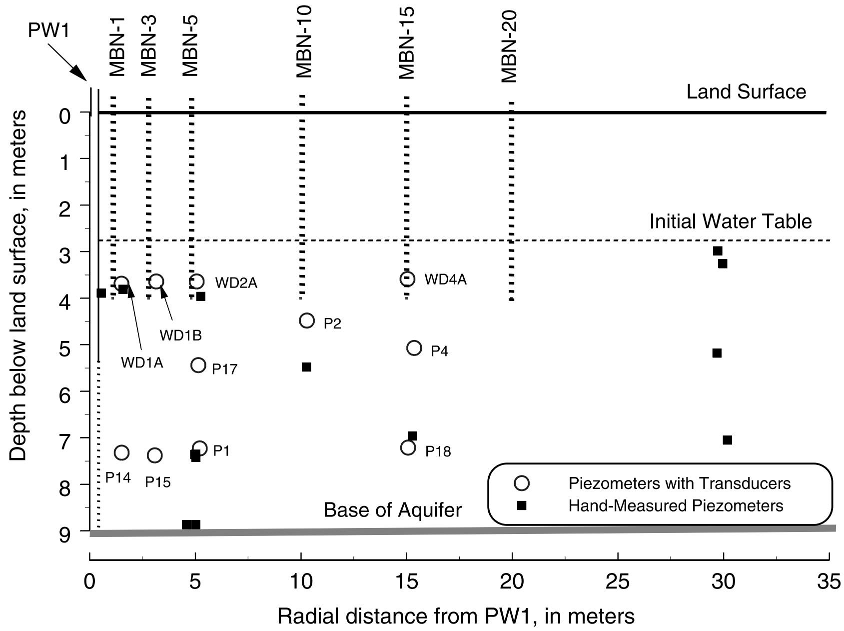

Figure 2. Vertical section of the aquifer at the Borden test site showing the positions of the pumped well screen (dashed vertical line at the bottom of PW1), the labeled piezometers with transducers, the piezometers measured by hand, and the neutron access tubes.

[11] Table 1 provides a listing of all observation piezometers with usable drawdown data, including their radial distances from the pumped well, X and Y coordinates, depths below the initial water table, and drawdowns after pumping has stopped (for general reference). In Table 1, PW1 refers to the pumped well, WD and WS mean deep and shallow water table wells, respectively, and P means piezometer. Locations of the piezometers and other information in Table 1 were obtained from *Bevan* [2002] and M. J. Bevan (written communication, March 2003). In Table 1 the piezometers are divided into two groups: one measured by means of transducers (in the order presented by Bevan) and the other measured (twice daily) by hand. The drawdown data were taken from *Bevan* [2002, Appendix B]. As indicated in Table 1, drawdowns in two of the piezometers WS7A and WS7 located 30 m from the pumped well were inadvertently omitted from the analysis. Because measured drawdowns in WS7A and WS7 were nearly identical to the measured drawdowns in P5 and P6, also located 30 m from the pumped well, their omission likely had negligible effect upon the analysis.

[12] At the start of the test the water table (defined as the pore fluid pressure that is equal to atmospheric pressure) was located at a depth of about 2.75 m below the nearly horizontal land surface, leaving an initial saturated thickness of 6.25 m. The pumped well is screened over the bottom 3.65 m of the aquifer and has an internal diameter (ID) of 0.13 m. According to *Bevan et al.* [2005] most observation piezometers are screened over a length of 0.35 m. Absent specific information to the contrary, it is assumed herein that all piezometers are screened over a length of 0.35 m. Piezometers used by *Nwankwor et al.* [1984] and *Nwankwor* [1985] are all reported to have IDs of 0.035 m. *Bevan* [2002] reports a range of piezometer diameters, the maximum of which is 0.050 m. The diameters of transducer-measured piezometers used in this paper are presumed to be 0.050 m unless indicated otherwise (see Table 1).

[13] For monitoring soil moisture profiles, six neutron access tubes composed of 0.05 m ID PVC pipes (closed at the bottom) were installed to a depth of 4.0 m below land surface at approximate radial distances of 1, 3, 5, 10, 15, and 20 m from the pumped well. Logging was performed in each access tube prior to, during, and after the aquifer test (to monitor recovery) over depths of 1.25–3.5 m below ground surface with measurements at 0.05 m intervals. The sampling frequency in time and space defined vertical variations in the moisture profile at sufficiently high resolution to support detailed modeling. During pumping, soil moisture measurements were made at frequent intervals from 480 min to just before pumping ceased at 10,560 min.

[14] Well PW1 was pumped at a constant discharge rate of 40 L/min for 7 d. The pumping rate was monitored at regular intervals by noting the time required to fill a container of known volume. After pumping stopped, drawdown and soil moisture measurements continued to be made for an additional 5 d in order to monitor recovery. Analysis of the recovery cycle is beyond the paper's scope. It is recognized, however, that analysis of the recovery cycle is ultimately required for a thorough analysis of the processes occurring in response to the aquifer test.

3. Analyses of the Aquifer Test

3.1. Analytical Approaches

[15] In order to estimate saturated zone hydraulic and geometric characteristics at the Borden site, two analytical models for flow to a well in an unconfined aquifer are used: (1) the model described by *Moench et al.* [2001] and (2) the model described by *Mathias and Butler* [2006]. The *Moench et al.* [2001] model, summarized by *Moench*

Table 1. Locations and Depths of the Pumped Well and Observation Piezometers[a]

Well Number	Radial Distance,[b] m	X Coordinate	Y Coordinate	Depth,[c] m	Drawdown at End of Pumping, m
		Transducer-Measured Wells			
PW1	0.065	0.00	0.00	2.60[d]	3.0
WD1A	1.51	−0.39	−1.46	0.94	0.605
WD1B	3.15	−0.29	−3.14	0.89	0.567
WD2A	5.07	−0.79	−5.01	0.89	0.507
WD4A	15.05	−0.53	−15.04	0.84	0.296
P14	1.51	0.10	−1.51	4.57	1.005
P15	3.08	0.23	−3.07	4.63	0.746
P17	5.15	−0.16	−5.14	2.69	0.529
P1[e]	5.22	0.21	−5.21	4.48	0.551
P2[e]	10.28	0.74	−10.26	1.73	0.382
P18	15.07	−0.25	−15.07	4.46	0.295
P4[e]	15.36	0.89	−15.34	2.32	0.311
		Hand-Measured Wells			
WD1	1.59	0.39	−1.54	1.06	0.630
WD2	5.27	0.60	−5.23	1.21	0.510
WD20	0.55	0.00	0.55	1.14	0.650
WS7[f]	29.74	1.07	−29.72	0.23	0.155
WS7A[f]	29.97	−0.53	−29.96	0.51	0.155
P5	30.19	0.55	−30.19	4.30	0.155
P6	29.69	0.00	−29.69	2.43	0.150
P3	15.28	0.46	−15.27	4.21	0.300
P22	10.27	10.08	1.96	2.73[g]	0.390
P13	4.59	−3.06	3.42	6.10[g]	0.705
P20	4.97	−0.46	4.95	4.61	0.580
P16	5.02	−0.38	−5.01	6.10	0.650
P19	5.02	5.02	−0.13	4.60	0.600
P21	5.04	−5.03	0.18	4.67[g]	0.560

[a]See Figures 1 and 2. Also shown are measured drawdowns at end of the test.
[b]Distance from center of pumped well.
[c]Depth below the initial water table to the center of the 0.35 m screen.
[d]Depth below the initial water table to the top of the pumped-well screen.
[e]Inner diameter of 0.035 m.
[f]Drawdown data was inadvertently omitted from the analysis.
[g]Screen length unknown but assumed to be 0.35 m.

[2004], is referred to in this paper as the Moench model. The Moench model is designed to account for transient two-dimensional, axisymmetric flow toward a partially penetrating pumped well of finite diameter in a compressible, anisotropic and homogeneous aquifer. Included in the model is storage in the pumped well, skin at the pumped-well screen, and delayed response of observation piezometers. The model does not include unsaturated zone hydraulic characteristics but does allow for gradual drainage from the zone above the water table. This is accomplished with the introduction in the water table boundary condition a finite series of exponential terms. *Moench* [2004] describes how the proposed boundary condition gives rise to improved estimates of aquifer hydraulic characteristics, including specific yield, hydraulic conductivity, and saturated thickness and, hence, improves matches between measured and simulated drawdowns. Details pertaining to the mathematical model are available elsewhere [*Moench*, 1997; *Barlow and Moench*, 1999; *Moench et al.*, 2001].

[16] The model proposed by *Mathias and Butler* [2006] accounts for transient two-dimensional, axisymmetric, saturated zone flow toward a fully penetrating well of infinitesimal diameter in a compressible, anisotropic and homogeneous aquifer. By using soil moisture and RHC relations that are represented by simple exponential functions, the model simulates vertical (but not horizontal) flow in a zone of finite thickness above the top of the capillary fringe (or air entry pressure head). *Mathias and Butler* [2006] derive a Laplace transform drainage function on the basis of a linearized Richards equation that can be incorporated into existing Laplace transform solutions for flow to a well in an unconfined aquifer [e.g., *Moench*, 1997] so that effects of partial penetration, wellbore storage and skin, delayed piezometer response, etc. can be included in an analysis.

[17] In this paper, aquifer parameters are estimated with a nonlinear parameter estimation algorithm PEST. (The use of this product does not imply endorsement by the U.S. government.) To reduce the computational effort, six values of drawdown for each log cycle of time are taken from the available measured drawdowns for each of the piezometers with transducers. All available hand-measured drawdowns are used. This amounts to a total of 428 drawdown values for the piezometers and pumped well listed in Table 1. For purposes of parameter estimation all drawdown values are weighted equally. The PEST algorithm determines parameter values that result in a minimization of squared differences between measured drawdowns and drawdowns simulated with the specified model. One measure of this minimization or "goodness of fit" is the root-mean-square error (RMSE);

Table 2. Parameters Estimated for the *Moench* [2004] Model Using the *de Hoog et al.* [1982] Algorithm in the WTAQ4 Code[a]

Parameter	Estimated Value	95% Confidence Limits: Lower Limit	95% Confidence Limits: Upper Limit	Initial Value
S_w	1.66	1.56	1.76	2.0
S_s, m^{-1}	4.45E-5	2.45E-5	8.09E-5	1.E-3
S_y	0.250	0.218	0.286	0.1
b, m	6.23	5.79	6.67	10.
K_z, m/s	3.05E-5	2.74E-5	3.41E-5	1.E-4
K_r, m/s	6.70E-5	6.13E-5	7.33E-5	1.E-4
α_1, min^{-1}	1.65E-4	5.15E-5	5.26E-4	1.E-5
α_2, min^{-1}	6.90E-3	2.75E-3	1.73E-2	1.E-3
α_3, min^{-1}	4.42E-2	2.33E-2	8.37E-2	1.E-1

[a]The root-mean-square error is 0.0228 m. Read 4.45E-5 as 4.45×10^{-5}.

another is the upper and lower 95% confidence limits generated by PEST.

3.1.1. Analysis With the Moench Model

[18] The aquifer test parameters are estimated using a revised version of WTAQ3 [*Moench*, 1997], called WTAQ4 in this paper. WTAQ4 allows for numerical inversion of the Laplace transform solution by the *de Hoog et al.* [1982] algorithm. The results of the analysis are shown in Table 2. The parameters in Table 2 are essentially unchanged from the values obtained using the *Stehfest* [1970] algorithm, which is the method of numerical inversion used in WTAQ3 and in the USGS model WTAQ [*Barlow and Moench*, 1999]. In Table 2, S_w is the dimensionless wellbore skin factor defined by *Moench* [1997], S_s is the specific storage, S_y is the specific yield, b is the saturated thickness, K_r is the saturated hydraulic conductivity in the horizontal direction, and K_z is the saturated hydraulic conductivity in the vertical direction. S_w is the ratio of horizontal hydraulic conductivity of the aquifer to hydraulic conductivity of the skin multiplied by the ratio of skin thickness to well radius [*Moench*, 1997]. The wellbore skin factor S_w is needed to account for the possibility of large drawdown in the pumped well compared with small drawdown in the aquifer adjacent to the pumped well and has a small but significant influence on the estimated value of specific storage S_s. Also shown in Table 2 are the initial values used and the 95% confidence limits that result from the estimation process. The parameters α_1, α_2, and α_3 are a set of empirical fitting parameters included in the water table boundary condition of the model that are designed to improve the match between measured and simulated drawdown in the intermediate-time range, which is the range most influenced by drainage from the zone above the water table [*Moench*, 2004, equation (2)].

[19] The estimated value of S_s shown in Table 2 is deemed reasonable for the unconsolidated, granular materials found at the Borden site. Although the value of S_s in Table 2 is smaller, as it should be, than values generally obtained from aquifer test analyses that ignore effects of wellbore storage, skin, and delayed piezometer response, it is still relatively large compared with values obtained from extensometer and earth-tide response studies. *Heywood* [1995] reports values of 7.3×10^{-6} to 1.1×10^{-5} m^{-1} for S_s in the upper 100 m of an unconfined aquifer within the Hueco basin of the Rio Grande valley, Texas. The numbers suggest that inelastic storage present in the younger and very shallow, unconsolidated materials at the Borden site may be the cause for the relatively large estimated value.

[20] The estimated value of S_y in Table 2 is also deemed reasonable for the granular materials at the Borden site and is large compared with values often obtained by aquifer test analyses that that assume instantaneous drainage from the unsaturated zone. (Assuming instantaneous drainage from the unsaturated zone, the estimated value of specific yield using the Moench model is approximately 0.18.) The estimated saturated thickness b in Table 2 is nearly identical to the value known from stratigraphy given by *Bevan et al.* [2005] for the August 2001 test.

[21] Figure 3 shows comparisons between measured and simulated drawdowns using the parameters given in Table 2. The comparisons show reasonably good agreement considering the local heterogeneity that exists at the site [*Bevan et al.*, 2005]. The largest differences between measured and simulated responses occur in piezometers WD1A, P1, P2, and P18, and even in those piezometers there appears to be good agreement in the late time range.

[22] The hand-measured drawdowns in piezometers P19, P20, and P21 and the late time transducer response of piezometer P1 are very close to one another. Table 1 shows all four piezometers of these piezometers to be located at roughly the same depth and radial distance from the pumped well, but their azimuths are displaced 90 degrees from one another at approximately the four points of the compass (M. J. Bevan, written communication, March 2003), suggesting azimuthal homogeneity at a depth of about 4.6 m below the initial water table.

[23] *Bevan et al.* [2005] point out that there is a slowing (seen on semilog plots) in the rate of change of drawdown in observation piezometers after 5800 min that they attribute to possible interaction with a distant recharge boundary. There is also a gradual decline, from 3.0 to 2.9 m, in the drawdown measured at the pumped well after 5800 min that might be attributed to a declining pumping rate. It is of interest to compare the parameters in Table 2 of this paper with those of *Endres et al.* [2007, Table 2] for the same aquifer test using the WTAQ model but with a data set that included the 11 piezometers with transducers but no hand-measured piezometers and 6000 min of pumping. The parameters obtained by *Endres et al.* [2007] are, except for the specific storage, similar to those in Table 2 but have a smaller range in the 95% confidence limits, probably as a consequence of the reduced data set. Their estimate of specific storage is larger than that shown in Table 2 and would appear to be the consequence of neglecting delayed piezometer response and wellbore skin effects.

3.1.2. Analysis With the Model of *Mathias and Butler* [2006]

[24] WTAQ4 was coded to incorporate the *Mathias and Butler* [2006] model in the manner described by the authors, recognizing the differences in model notation. *Mathias and Butler* [2006] follow the work of *Kroszynski and Dagan* [1975] extending their solution to allow for elastic storage in the saturated zone, different soil moisture retention and RHC functions, and unsaturated zone of finite thickness. The solution also differs from that of *Kroszynski and Dagan*

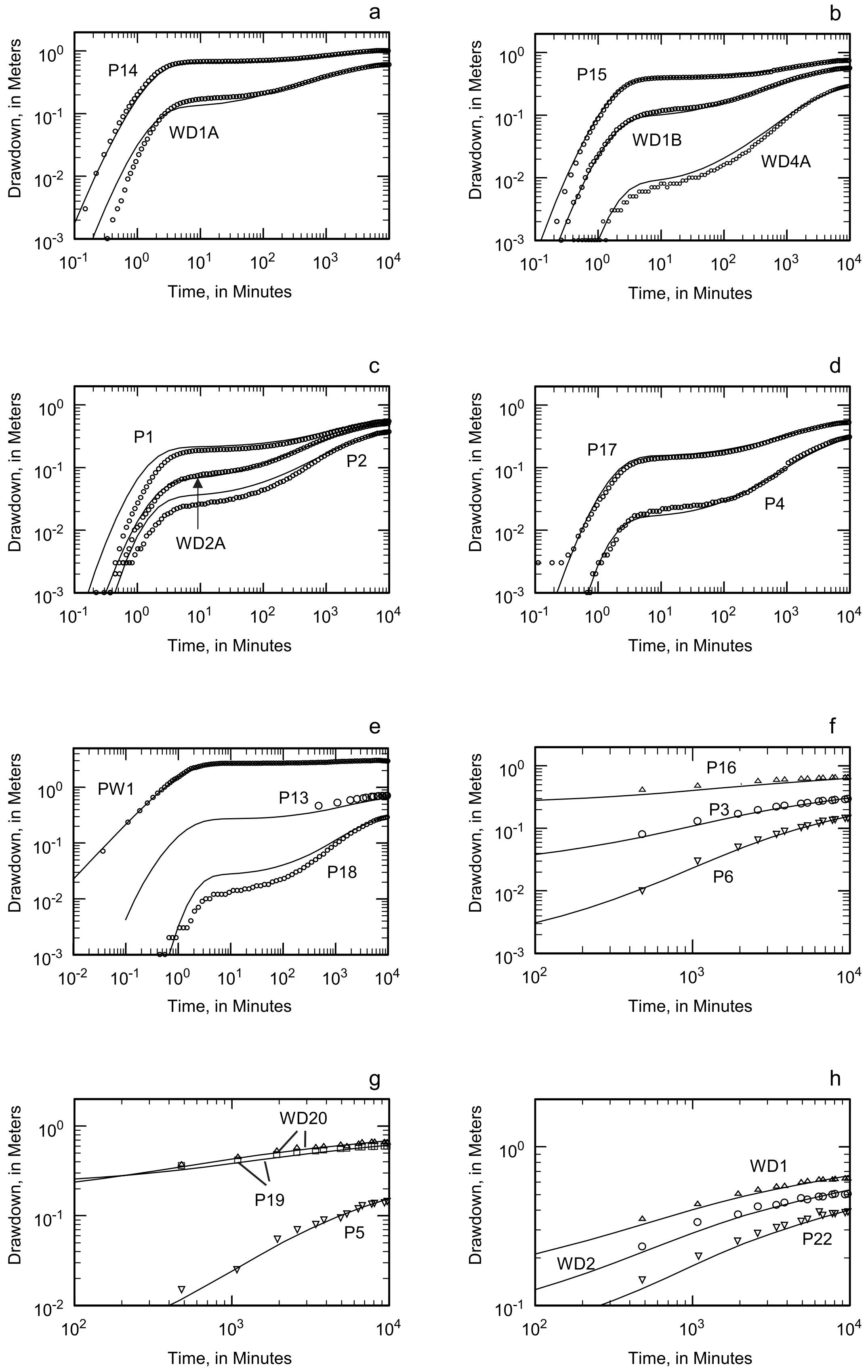

Figure 3. Comparisons of measured drawdowns (open symbols) for piezometers listed in Table 1 with theoretical responses (solid lines) using the parameters in Table 2 and the model of *Moench et al.* [2001]. (Piezometers P20 and P21 are omitted from the figure because their measured and simulated drawdowns are the same as those of P19.)

[1975] in that *Mathias and Butler* [2006] assume for simplicity and improved analytical tractability that only vertical flow occurs in the unsaturated zone above the top of the capillary fringe. The analytical approximations used for effective saturation $S_e(h_c)$ and RHC $k_{rel}(h_c)$ for $h_c < h_b$, as proposed by *Gardner* [1958], are written as follows:

$$S_e(h_c) = e^{a_c(h_c - h_b)} \qquad h_c < h_b \tag{1}$$

$$k_{rel}(h_c) = e^{a_k(h_c - h_b)} \qquad h_c < h_b \tag{2}$$

where $S_e(h_c) = (\theta - \theta_r)/(\phi - \theta_r)$, θ is the volumetric moisture content, θ_r is the residual moisture content, ϕ is the porosity, h_c is the capillary pressure head ($h_c < 0$), h_b is the air entry (or bubbling) pressure head ($h_b < 0$), K_{rel} is the ratio of unsaturated hydraulic conductivity to saturated hydraulic conductivity, a_c is the moisture retention exponent, and a_k is the RHC exponent. Recently, *Ghezzehei et al.* [2007] have shown that the Gardner RHC can be recast in terms of the *van Genuchten* [1980] RHC function through the use of a simple conversion formula that is valid in the midrange of saturation, thereby enhancing the applicability of analytical models that use equation (2).

[25] Air entry pressure head h_b is not included in the mathematical solution derived by *Mathias and Butler* [2006] as it was eliminated in the necessary process of linearizing the nonlinear boundary condition at the top of the capillary fringe. The air entry pressure head must be determined independently of the analytical model for use in displaying the thickness of the capillary fringe.

[26] In this analysis of the Borden site aquifer test it is assumed for simplicity that the unsaturated zone is infinitely thick even though the *Mathias and Butler* [2006] model allows for finite thickness. The assumption is not unreasonable as volumetric soil moisture measurements show the land surface to be well above the zone of transition from the top of the capillary fringe at full saturation to the volumetric water content at residual saturation (see Figure 8).

[27] Figure 4a shows a plot of the measured "background" volumetric moisture contents above the initial water table for the access tube MBN-5 (solid squares). By visual inspection, allowing for measurement variability, it is estimated that the air entry pressure head h_b is -0.40 m. (See Figure 6 for measured background volumetric moisture contents above the initial water table for all access tubes.) By visual inspection, an approximate fit to equation (1) was found with $a_c = 5.0\ \mathrm{m}^{-1}$ and is shown in Figure 4a as a dotted line.

[28] Tables 3a and 3b show the estimated parameters obtained using WTAQ4. The WTAQ4 code includes all the aquifer and wellbore features used in estimating the

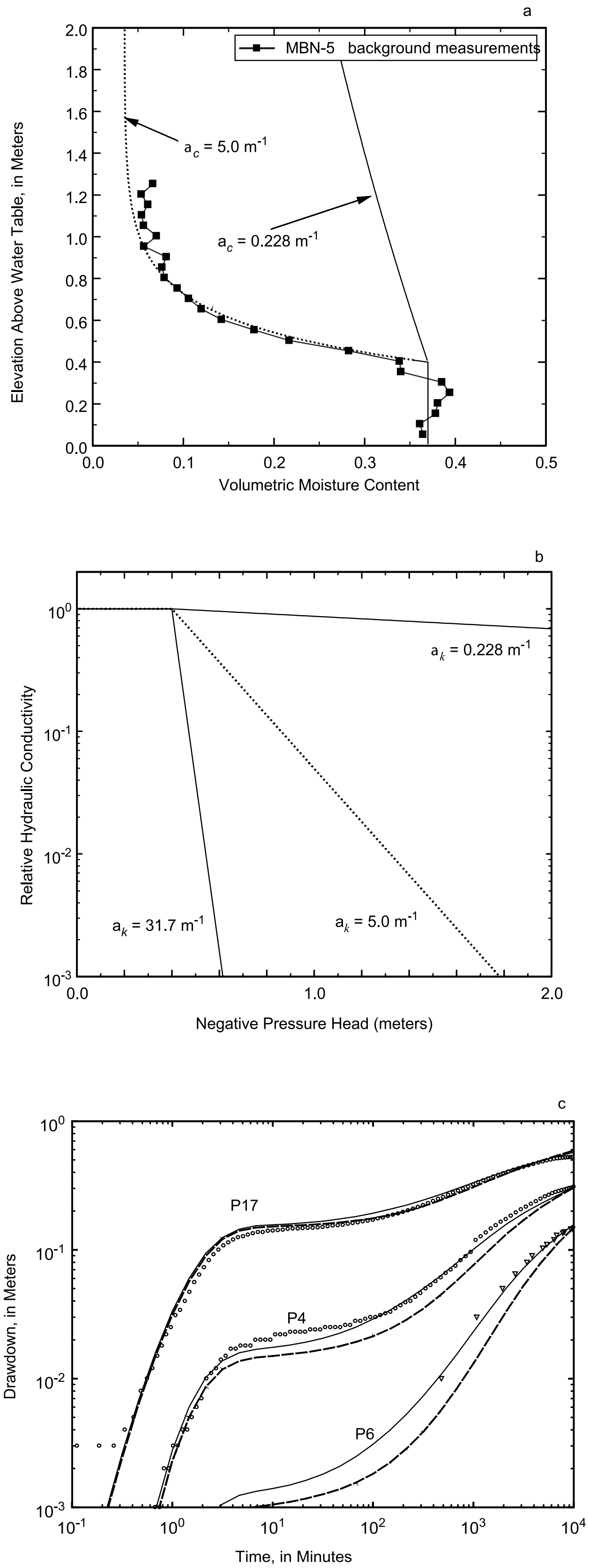

Figure 4. Unsaturated zone characteristics and results obtained with the *Mathias and Butler* [2006] model: (a) measured background soil moisture in MBN-5 compared with the simulated distributions using equation (1) with the indicated values of the moisture retention exponent, (b) relative hydraulic conductivity versus negative pressure head using equation (2) with the indicated values of the relative hydraulic conductivity exponent, and (c) drawdowns and simulations in the indicated piezometers from Figures 3d and 3f (solid lines) compared with drawdowns simulated with the Mathias and Butler model under the assumption $a_c = a_k = 5.0\ \mathrm{m}^{-1}$ (dashed lines).

Table 3a. Parameters Estimated for the *Mathias and Butler* [2006] Model With $a_c = 5.0\ m^{-1}$ Using WTAQ4[a]

Parameter	Estimated Value	95% Confidence Limits		Initial Value
		Lower Limit	Upper Limit	
S_w	1.74	1.63	1.86	2.0
S_s, m^{-1}	3.76E-5	1.87E-5	7.54E-5	1.E-3
b, m	6.20	5.79	6.62	10.
K_z, m/s	2.90-5	2.64E-5	3.19E-5	1.E-4
K_r, m/s	6.84E-5	6.32E-5	7.41E-5	1.E-4
a_k, m^{-1}	31.7	27.2	37.0	100.

[a]The root-mean-square error is 0.0248 m. Read 3.76E-5 as 3.76×10^{-5}.

parameters in Table 2 but with drainage from the unsaturated zone described by the *Mathias and Butler* [2006] model rather than the Moench model. To obtain the estimated parameters in these tables, it was necessary to hold the specific yield S_y constant (at the value given in Table 2). This is because S_y was found to be highly correlated with the relative conductivity exponent a_k resulting in an unrealistic estimated value ($S_y \sim 0.65$) and an excessively large spread in the 95% confidence limits for both S_y and a_k. The parameters in Table 3a result from using a fixed value for the moisture retention exponent ($a_c = 5.0\ m^{-1}$) and an adjustable RHC exponent a_k. The parameters in Table 3b result from the assumption $a_c = a_k$ used by *Kroszynski and Dagan* [1975] and by *Tartakovsky and Neuman* [2007].

[29] The aquifer parameter values shown in Tables 3a and 3b are consistent with those in Table 2. Drawdowns simulated with the parameters in Tables 3a and 3b vary only slightly when compared with those shown in Figure 3 for the parameters in Table 2. The root-mean-square error (RMSE) obtained for the parameters in Table 2 (0.0228 m) indicates that the fit with three adjustable empirical parameters for drainage is marginally better, as it should be, than with a single adjustable drainage parameter a_k in Table 3a (0.0248) or $a_c = a_k$ in Table 3b (0.0255). Of course, having a representation of unsaturated zone hydraulic properties is a major advantage over models that do not account for unsaturated zone processes.

[30] Figures 4a and 4b illustrate the behavior of the estimated unsaturated zone hydraulic properties in Tables 3a and 3b. The retention curve for $a_c = 0.228\ m^{-1}$ in Figure 4a shows that use of the same exponential relation for both soil moisture and relative hydraulic conductivity (i.e., $a_c = a_k$) yields a soil moisture retention that departs radically from the measured data. Extrapolation of the solid line (labeled $a_c = 0.228\ m^{-1}$) to the land surface, 2.75 m above the initial water table, yields a water saturation of about 60%. Also, on the basis of the assumption $a_c = a_k$, the RHC is seen in Figure 4b to decline only slightly with elevation (increasing negative pressure head) above the top of the capillary fringe. With a_c held constant at $5.0\ m^{-1}$ and $a_c \neq a_k$, Figure 4b shows that the RHC declines steeply ($a_k = 31.7\ m^{-1}$) with increasing elevation above the top of the capillary fringe. (The results are similar to those presented by *Mathias and Butler* [2006, Figure 5] on the basis of a rather limited data set from another test conducted in the same aquifer by *Nwankwor et al.* [1984].)

[31] To further illustrate the consequences of making the assumption $a_c = a_k$, a run was made with $a_c = a_k = 5.0\ m^{-1}$. Figure 4b shows the RHC curve for $a_c = 5.0\ m^{-1}$. The result was a set of estimated parameters similar to those shown in Tables 3a and 3b but with an enlarged RMSE (0.0296 m). (The parameters S_s, K_z, and K_r were estimated to be $4.0 \times 10^{-5}\ m^{-1}$, 2.92×10^{-5} m/s, and 6.22×10^{-5} m/s, respectively.) Simulated drawdowns agree quite well with drawdowns measured in deep-seated piezometers close to the pumped well. However, they deviate significantly from measured drawdowns in the intermediate-time range, especially in piezometers located near the water table or at large distances from the pumped well (10 m or greater). Figure 4c compares drawdowns simulated under this assumption with measured and simulated drawdowns in piezometers seen in Figures 3d and 3f. Piezometers P17, P4, and P6 are located at distances of 5, 15, and 20 m, respectively, from the pumped well and at approximately equal depths (2.5 m) below the initial water table. Figure 4c illustrates the importance of piezometers located at large distances from the pumped well for obtaining a complete and accurate representation of aquifer properties.

[32] These findings demonstrate that it is necessary to have different exponential representations for the soil moisture distribution and RHC. Results show the model assumption $a_c = a_k$ serves only as a means for fitting simulated drawdown to measured drawdown. The estimated parameter ($a_k = 0.228\ m^{-1}$) yields totally unrealizable soil moisture retention and casts doubt upon the validity of the assumption $a_c = a_k$ used in the analytical models of *Kroszynski and Dagan* [1975] and *Tartakovsky and Neuman* [2007], apparently for reasons of mathematical tractability. These models do have the advantage of allowing for components of horizontal flow in the unsaturated zone. It remains to be determined, however, whether horizontal flow in the unsaturated zone above the capillary fringe is important for parameter estimation. The numerical model VS2DT to be discussed in the next section allows for horizontal and vertical flow in both the unsaturated and saturated zones and should help to assess the importance of horizontal flow above the capillary fringe for parameter estimation (see section 4).

3.2. Numerical Approaches

3.2.1. Numerical Approach Using the *Brooks and Corey* [1964] Model in VS2DT

[33] In order to estimate unsaturated zone characteristics at the Borden site with fewer constraints on how flow in the unsaturated zone is represented, a 2-D axisymmetric (r-z) numerical model for variably saturated flow VS2DT [*Healy*, 1990; *Lappala et al.*, 1987] was used. The VS2DT model has various options for specification of unsaturated zone

Table 3b. Parameters Estimated for the *Mathias and Butler* [2006] Model With $a_c = a_k$ Using WTAQ4[a]

Parameter	Estimated Value	95% Confidence Limits		Initial Value
		Lower Limit	Upper Limit	
S_w	1.51	1.43	1.61	2.0
S_s, m^{-1}	3.32E-5	1.50E-5	7.36E-5	1.E-3
b, m	6.21	5.82	6.60	10.
K_z, m/s	3.21E-5	2.90E-5	3.55E-5	1.E-4
K_r, m/s	6.54E-5	6.07E-5	7.05E-5	1.E-4
a_k, m^{-1}	0.228	0.177	0.295	1.0

[a]The root-mean-square error is 0.0255 m. Read 3.32E-5 as 3.32×10^{-5}.

Table 4. Parameters Estimated for the *Brooks and Corey* [1964] Model Using VS2DT[a]

Parameter	Estimated Value	95% Confidence Limits: Lower Limit	95% Confidence Limits: Upper Limit	Initial Value
θ_r	0.030[b]	na[c]	na	0.030[b]
h_b, m	−0.330	−0.379	−0.281	−0.350
λ	0.435	0.382	0.494	2.5
K_z, m/s	3.11E-5	2.90E-5	3.32E-5	4.0E-5
K_r, m/s	6.60E-5	6.50E-5	6.71E-5	8.0E-5

[a]The root-mean-square error is 0.0246 m. Read 3.11E-5 as 3.11×10^{-5}.
[b]Fixed value.
[c]Na means not applicable.

characteristics. It can use one of several popular functional relations, alternative user-designed relations, or tabular measurements of volumetric moisture content and relative hydraulic conductivity.

[34] For the *Brooks and Corey* [1964] formulation, the relationship for soil moisture retention is

$$\begin{aligned} \theta &= \theta_r + (\phi - \theta_r)(h_b/h_c)^{\lambda} && h_c < h_b \\ \theta &= \phi && h_c \geq h_b \end{aligned} \tag{3}$$

and for relative hydraulic conductivity is

$$\begin{aligned} K_{rel} &= (h_c/h_b)^{-2-3\lambda} && h_c < h_b \\ K_{rel} &= 1 && h_c \geq h_b \end{aligned} \tag{4}$$

where λ is the pore size distribution index.

[35] The aquifer response was simulated using the aquifer geometric configuration described previously and with the piezometer specifications given in Table 1. A graphical user interface described by *Hsieh et al.* [2000] was used to facilitate the initial design of the numerical aquifer test model. The outer boundary was defined as a no-flow boundary and was placed at a radial distance of 100 m from the pumped well. This was sufficiently far that it had no perceptible influence upon drawdown in any of the piezometers during the test. The fixed grid was laid out with 152 rows and 58 columns so that there was a fine spacing (∼2–3 cm) of rows from 1.3 m above to 0.7 m below the initial water table, a fine spacing of rows (∼3 cm) around the top of the pumped well screen, and a fine spacing (4–40 cm) of columns from 0.1 to 3.0 m from the pumped well in regions of rapidly changing hydraulic head and changing volumetric moisture content. At increased distances the spacing was increased to a maximum of 15 m near the outer boundary of the model domain. Tests were conducted to assess the suitability of the grid dimensions and time step size to accurately simulate the hydraulic head distribution and were found to be satisfactory. Essentially identical drawdowns were obtained using 73 columns as were obtained using 58. Small differences were apparent in the simulated soil moistures, but these do not significantly change the shape of the simulated soil moisture distribution (see Figure 10). Reductions in the maximum allowable time step made no noticeable difference in computer output. The pumped well was modeled to fit the described specifications as closely as possible and includes a layer (∼0.06 m in thickness) of skin around the well screen. The hydraulic conductivity of the skin was estimated as described below. The interior of the well was assigned a very large (essentially infinite) hydraulic conductivity and a large specific storage (S_s = 1 m^{-1}) in the region above the water level in the well. The top of the pumped well was a constant discharge boundary.

[36] The parameter estimation algorithm PEST was used in combination with VS2DT to minimize the squared differences between measured and computed drawdowns in the pumped well and in 23 observation piezometers. As with the analytical models, to reduce computation time, about six values per log cycle of time were used for piezometers with transducers and all measured values were used for hand-measured piezometers, each with equal weighting. Unlike the analytical models, the location of the base of the aquifer in the numerical model was, of course, not easily adjustable because of the fixed nodal arrangement. Also, as the diameter and length of the observation piezometers were not included in the numerical model, the specific storage S_s was not estimated and was instead set equal to the value obtained analytically (see Table 2). Because the numerical model does not allow for delayed piezometer response, drawdown measurements made prior to the first 6 min were not included in the optimization. Assigning zero weight to these data is likely to have negligible influence on estimation of the remaining parameters. The hydraulic conductivity of the skin surrounding the screen of the pumped well was determined independently using the measured drawdown in the pumped well and the analytically estimated hydraulic conductivity of the aquifer (Table 2). The hydraulic conductivity of the skin was estimated to be 1.8×0^{-5} m/s.

[37] Only five parameters in the numerical model using the Brooks and Corey functional relations remained to be adjusted: vertical and horizontal hydraulic conductivity in the saturated zone, residual moisture content, air entry pressure head, and pore size distribution index. Results are shown in Table 4. In spite of differences in methodology, the horizontal and vertical hydraulic conductivities in Tables 2, 3a, 3b, and 4 are in close agreement. Because of strong nonlinearities in flow in the unsaturated zone, and to obtain satisfactory convergence, it was found necessary to fix the value of residual moisture content θ_r. Initial values of the horizontal and vertical hydraulic conductivity were

Table 5. Approximate Thicknesses of the Capillary Fringe at Six Distances From the Pumped Well and at Different Times as Determined From Measured Soil Moisture Profiles and the Location of the Water Table as Simulated by VS2DT[a]

Access Tube Name	Radial Distance, m	Approximate Thickness of Capillary Fringe, m: t = 0	t = 480, min	t = 2220, min	t = 10,560, min
MBN1	1.09	0.34	0.50	0.53	0.58
MBN3	2.78	0.33	0.45	0.52	0.55
MBN5	4.81	0.35	0.44	0.48	0.54
MBN10	9.99	0.35	0.40	0.46	0.52
MBN15	14.99	0.34	0.39	0.45	0.50
MBN20	19.96	0.41	0.41	0.46	0.48
Average		0.35	0.43	0.48	0.53

[a]See Figure 8.

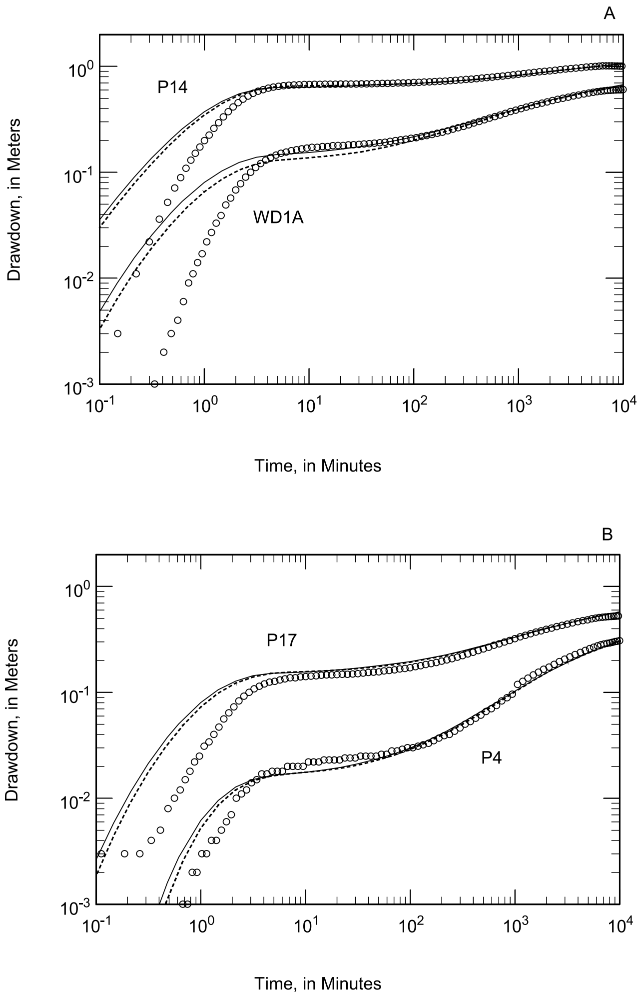

Figure 5. Numerical simulations of drawdown (solid lines) compared with transducer-measured drawdown in four piezometers (open circles). Also shown are analytical simulations of the drawdowns wherein delayed piezometer response is omitted (dashed lines).

chosen that were reasonably close to the estimates on the basis of the analytical analyses. The initial value of the air entry pressure head h_b was based on background soil moisture measurements for all neutron access tubes (see Table 5) and the known elevation of the water table just prior to the start of the pumping. The value of h_b could just as well have been fixed at -0.35 m without significantly changing the other parameters or the RMSE. Extensive numerical experimentation has shown h_b to be only weakly sensitive to variation in λ, K_z, or K_r and, therefore, was assumed fixed for purposes of subsequent numerical analyses.

[38] Drawdowns simulated by VS2DT under the parameters in Table 4 are not shown (except in Figure 5) as they are essentially indistinguishable, in the intermediate and late time ranges, from the drawdowns simulated analytically (Figure 3). The simulated drawdowns deviate from the analytical results in Figure 3 in the early time range because effects of delayed piezometer response were not simulated with the VS2DT model. Figure 5 shows measured and numerically simulated drawdowns (solid lines) in four piezometers with transducers using the parameters in Table 4. Figure 5 also shows drawdowns simulated analytically (dashed lines) using the parameters in Table 2 but omitting effects of delayed piezometer response. Since it is early time data that are needed for estimates of specific storage S_s, results show the necessity of including delayed piezometer response in the parameter estimation analysis. This is easily accomplished analytically. To do so numerically would require detailed numerical description of each of the piezometers. Alternatively, an approach similar to that described by *El-Kadi* [2005] could be used, but an additional step would be required; namely, convolving the early time numerical output with the analytical expression proposed by *El-Kadi* [2005, equation (11)].

[39] Figure 6 shows a plot of measured background volumetric moisture contents versus elevation above the initial water table for each of the six neutron access tubes. Also shown is a Brooks and Corey retention curve ($\lambda = 2.5$) that was obtained by fitting equation (3) to the measured background volumetric moisture content above the initial water table at neutron access tube MBN-5. The pore size distribution index that was found is consistent with the value of $\lambda = 2.48$ obtained by *Kueper and Frind* [1991], on the basis of seven samples of Borden aquifer material.

[40] Figure 6 also shows the retention curve based on the estimated parameter values given in Table 4. Note that the retention curve in Figure 6 for $\lambda = 0.435$ is similar to the retention curve in Figure 4a for $a_c = 0.228$ m^{-1} in that it bears no resemblance to the measured soil moisture data.

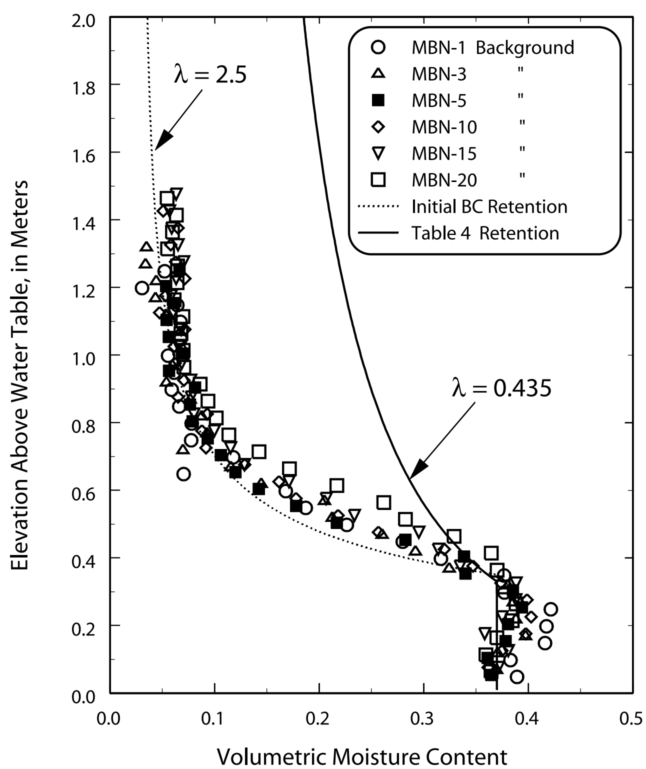

Figure 6. Measured background soil moistures compared with the initial retention curve (dotted line) and the estimated soil moisture retention (solid line) using the Brooks and Corey model and VS2DT with the parameters in Table 4.

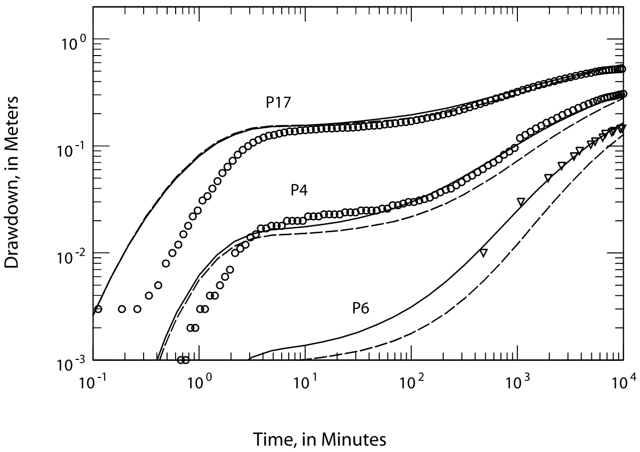

Figure 7. Drawdowns and simulations in the indicated piezometers using the parameters in Table 4 (solid lines) compared with drawdowns simulated with VS2DT using the Brooks and Corey model with $\lambda = 2.5$ (dashed lines).

Like the parameters in Table 3b, the parameters in Table 4 serve only as "fitting parameters." The Brooks and Corey functional relation as applied (using VS2DT) to the Borden site aquifer test is, therefore, no improvement over the analytical approach using the parameters in Table 3b or the approach using the Moench model and the parameters in Table 2.

[41] As with the analytical analysis wherein a run was made by forcing $a_c = a_k = 5.0$ m^{-1}, an additional run was made with the Brooks and Corey formulation and VS2DT by forcing λ to match the soil moisture observations. With λ fixed at 2.5, and $-h_b$ fixed at 0.35 m, the parameters K_z and K_r were estimated to be 3.14×10^{-5} m/s and 5.89×10^{-5} m/s, respectively, and the root-mean-square error RMSE became 0.0337 m. The enlarged RMSE demonstrates the overall degradation in the matches between measured and simulated drawdowns. Figure 7 shows comparisons of measured and simulated drawdowns in piezometers P17, P4, and P6 for this situation (λ fixed at 2.5). (As mentioned previously, piezometers P17, P4, and P6 are located at distances of 5, 15, and 20 m, respectively, from the pumped well and at approximately equal depths (2.5 m) below the initial water table.) The results are similar to the comparisons in Figure 4c except that because of effects of delayed piezometer response, early time data ($t < 6$ min) are not included in the parameter estimation (see Figure 5). As with the analytical model (wherein $a_c = a_k = 5.0$ m^{-1}), simulated drawdowns agree quite well in piezometers located near the pumped well but do not agree well at distant locations. Figure 7 (like Figure 4c) shows the importance of having piezometers located at distant points so that a complete and accurate representation of the aquifer properties can be obtained.

[42] Before proceeding with an alternative numerical model to resolve the problem with the *Brooks and Corey* [1964] formulation as used in VS2DT, it is necessary to quantify elongations of the capillary fringe described by *Bevan et al.* [2005].

3.2.2. Soil Moisture Measurements and Capillary Fringe Elongation Based on VS2DT

[43] Figure 8 shows volumetric moisture contents versus elevation at each of the six neutron access tubes at selected times of 480, 2220, and 10,530 min. Measured background volumetric moisture contents and initial elevation of the water table are also given in Figure 8. The water table elevations at the six neutron access tubes and at times of 480, 2220, and 10,560 min were obtained from VS2DT output using the parameters in Table 4. These water table elevations should be close to the actual field values in view of the agreement between measured and simulated drawdowns in the piezometers at intermediate and late times. The water table elevations in Figure 8 reflect the declining cone of depression around the pumped well and, when compared with the measured volumetric soil moisture values, show the elongation of the capillary fringe described by *Bevan* [2002] and *Bevan et al.* [2005]. *Bevan et al.* [2005, p. 57] point out that for volumetric moisture contents greater than 0.15, the moisture content profiles:

> . . .appear to translate downwards during pumping without significant changes in the character of that portion of the transition zone relative to the background profile.

Thus, the transition zone of the soil moisture profiles appears to translate vertically following the declining cone of depression but at a relatively slow rate, resulting in an increased elongation of the capillary fringe.

[44] To provide additional quantification of the capillary fringe elongation, expanded versions of Figure 8 were used to determine the thickness of the capillary fringe for each access tube location and at each of the selected times. The elevation of the top of the capillary fringe was determined by visually fitting a straight line through the (sometimes scattered) soil moisture data in the wet range of the transition zone, where $\theta = \sim 0.25$ to ~ 0.35, to a point where the line crosses the point of complete saturation (0.37). The thickness of the capillary fringe was then determined by subtracting the elevation of the water table as obtained from VS2DT and shown in Figure 8. The results and average values for each time of interest are provided in Table 5 and confirm the findings of *Bevan et al.* [2005, p. 52]:

> The capillary fringe extended significantly with pumping in comparison to its static thickness; this extension progressively increased during the entire test. The capillary fringe extension decreased with increasing radial distance and was inferred to have eventually extended out to between 20 and 25 m.

[45] *Bevan* [2002] and *Bevan et al.* [2005] show plots of the capillary fringe elongations, some of which differ from the results in Table 5 because of the methodology used to estimate the water table elevations and the capillary fringe elongations. *Bevan* [2002] estimated water table elevations by upward projection of drawdowns observed in nearby piezometers. Elongations of the capillary fringe were estimated from the elevation of a reference point within the zone of transition, taken as the moisture content of 0.22, compared with the position of the water table at each of the neutron access tubes.

[46] Results in Table 5 suggest that unsaturated zone hydraulic properties may be dependent on the rate of decline of the water table as it diminishes with time from the start of pumping and is greatest close to the pumped well. The decreased elongation of the capillary fringe with increasing radial distance from the pumped well and the continued elongation with increasing time, apparently without atten-

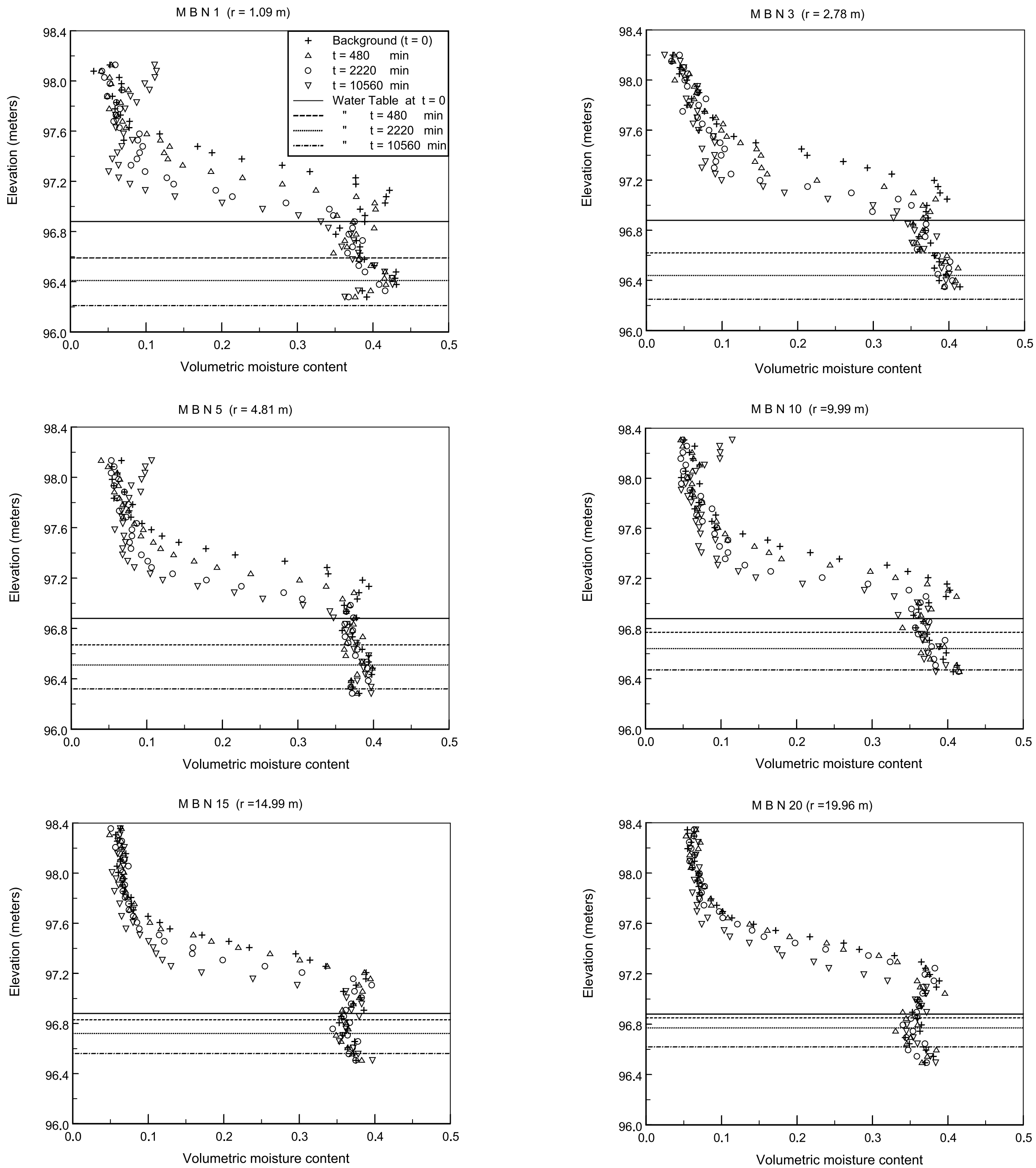

Figure 8. Measured volumetric moisture contents versus elevation just prior to the test (t = 0), at intermediate times (t = 480 min and t = 2220 min), and at end of test (t = 10,560 min) at the six neutron access tubes. Locations of the water table simulated with VS2DT using parameters in Table 4 are shown for times of 480, 2220, and 10,560 min. Volumetric moisture contents measured in MBN1, MBN5, and MBN10 at high elevations (near the land surface) resulted from a rainfall event prior to the end of the test.

uation, demonstrate a phenomenon that is not fully understood. Various mechanisms for the elongations are discussed in section 4.

[47] As explained at the beginning of this section, VS2DT was used to obtain the elevation of the water table at the locations and times of interest so that elongations of the capillary fringe could be estimated, resulting in Table 5. Unfortunately, it was not possible to use VS2DT to obtain estimates of the air entry pressure head with any degree of certainty as h_b was found to be only weakly sensitive to

Table 6. Parameters Estimated for *Assouline* [2001, 2004] Model Using VS2DT[a]

Parameter	Estimated Value	95% Confidence Limits: Lower Limit	95% Confidence Limits: Upper Limit	Initial Value
		RMSE = 0.0166 m and t = 480 min		
h_b, m	-0.43[b]	na[c]	na	-0.43[b]
η	4.97	4.13	5.99	3.0
K_z, m/s	3.76E-5	3.43E-5	4.12E-5	4.E-5
K_r, m/s	5.84E-5	5.69E-5	5.99E-5	8.E-5
		RMSE = 0.0218 m and t = 2220 min		
h_b, m	-0.48[b]	na	na	-0.48[b]
η	7.33	6.64	8.10	3.0
K_z, m/s	3.32E-5	3.07E-5	3.59E-5	4.E-5
K_r, m/s	6.31E-5	6.17E-5	6.45E-5	8.E-5
		RMSE = 0.0248 m and t = 10,560 min		
h_b, m	-0.53[b]	na	na	-0.53[b]
η	8.19	7.45	8.99	3.0
K_z, m/s	3.11E-5	2.91E-5	3.32E-5	4.E-5
K_r, m/s	6.62E-5	6.50E-5	6.73E-5	8.E-5

[a]The parameter $\lambda = 2.5$ for all runs. Read 3.76E-5 as 3.76×10^{-5}.
[b]Fixed value.
[c]Na means not applicable.

variation in the other hydraulic parameters and was assumed fixed at "known" values for purposes of the numerical analysis presented in the next section.

3.2.3. Numerical Approach Using the *Assouline* [2001] Model in VS2DT

[48] The analysis of the Borden site aquifer test data with VS2DT using the *Brooks and Corey* [1964] functional relations is shown above to be unsatisfactory whether the parameter λ was allowed to vary (giving rise to unrealizable soil moisture retention) or whether λ is fixed (giving a reasonable soil moisture retention but unsatisfactory matches between simulated and measured drawdowns). To help resolve the situation an alternative model was chosen that provides an estimable parameter for the RHC that differs from that used by *Brooks and Corey* [1964], analogous to what was achieved with the analytical approaches. The *Assouline* [2001] model, referred to in this paper as the Assouline model, establishes a relationship between relative hydraulic conductivity (RHC) and soil structure and texture through the parameter η and retains the pore size distribution index λ and air entry pressure head h_b used in the Brooks and Corey soil moisture retention function. The RHC derived by Assouline following the approach of *Mualem* [1976] is

$$
\begin{aligned}
K_{rel} &= (h_c/h_b)^{-\eta-\eta\lambda} \qquad && h_c < h_b \\
K_{rel} &= 1 && h_c \geq h_b
\end{aligned} \tag{5}
$$

Equation (5) is connected to equation (3) through λ and the ratio h_b/h_c and is inserted into VS2DT in this paper as a user-defined functional relation.

[49] The numerical model VS2DT is not designed to account for either temporal or spatially varying hydraulic parameters. Because the thickness of the capillary fringe is observed to vary in both time and space in the course of the aquifer test, simulations with VS2DT will necessarily involve approximations. The data set provided by *Bevan* [2002] is extensive so it is possible to treat the 7-d-long test as a set of two, three, or more tests of different duration. This makes it possible to observe changes in the controlling parameters with time, recognizing, of course, that at any particular time the parameters are a consequence of all prior measured data. Since spatial variation is not included in the model, only average values for the aquifer as a whole are obtained.

[50] Such an approach was taken here by assuming the existence of three tests lasting 480, 2220, and 10,560 min and by using VS2DT and the Assouline model for definition of the unsaturated zone characteristics. In this analysis elongation of the capillary fringe is considered to be "data" and is included in the analysis by using the "measured" average values in Table 5, which are assumed in the model to be equivalent to average values of $-h_{\mathrm{b}}$. Because h_b was found to be relatively insensitive to variation in the remaining hydraulic parameters, the average values were held constant in the estimation process. Results are shown in Table 6 wherein $\lambda = 2.5$ for all runs. The justification for the fixed value of $\lambda = 2.5$ is the essentially uniform downward vertical translation, seen in Figure 8, of the middle part of the transition zone of the soil moisture profile from a volumetric moisture content of about 0.15 to full saturation near the top of the capillary fringe. The adjustable parameters in Table 6 vary systematically as time increases; that is, η increases, K_z decreases, and K_r increases. This is probably as much a consequence of the incorporation of additional data into the estimation process at larger times as it is to the increase in the fixed values of h_b. The values of η in Table 6 are much larger than the values reported by *Assouline* [2001] for cores of various soil types. (Incidentally, effectively the same values of η were obtained by using an initial value of $\eta = 10$.) This suggests that effects of soil structure and texture, upon which the value of η is said to represent, depend on whether the model is applied at the local scale or field scale.

[51] Drawdowns simulated with VS2DT and the Assouline RHC using the parameters in Table 6 for $t = 10{,}560$ min, like those simulated with VS2DT and the Brooks and Corey RHC using the parameters in Table 4, compare well with measured drawdowns in the intermediate and late time range. (The early time range is not simulated.) The difference between the models lies in the fact that Assouline model provides an additional adjustable parameter in the RHC functional relation η that is independent of the parameter defining the soil moisture retention λ.

[52] Figure 9 shows comparisons of RHC versus negative pressure head for the value of λ obtained with the Brooks and Corey model (Table 4) and the value of λ and η obtained with the Assouline model (Table 6 with t = 10,560 min). Also shown is the RHC curve for the fixed value of λ (2.5) used with the Brooks and Corey model to simulate the unsatisfactory results seen in Figure 7. The numerical RHC parameters shown in Figure 9 (with a logarithmic abscissa) are analogous to the analytical RHC parameters shown in Figure 4b (with an arithmetic abscissa). Note that the RHC for the Assouline model in Figure 9 is numerically close to the RHC for the Mathias and Butler model in Figure 4b ($a_c = 31.7\ \mathrm{m}^{-1}$).

[53] As no soil moisture measurements were used in the estimation process (except to determine the fixed values of

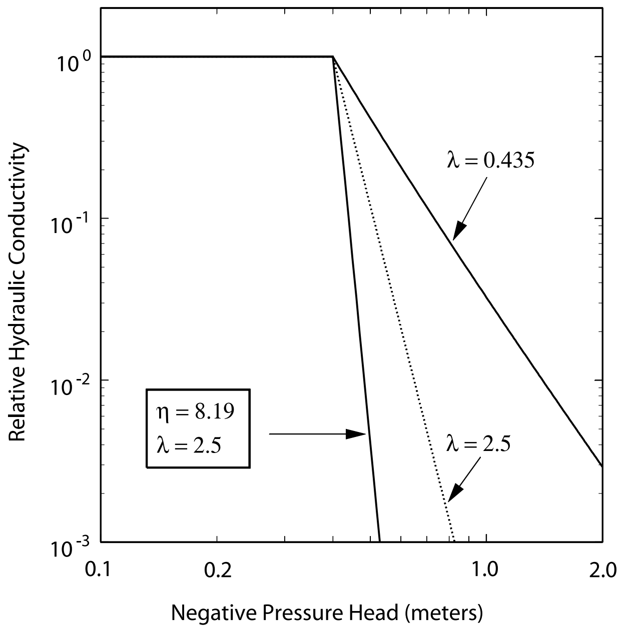

Figure 9. Relative hydraulic conductivity relations versus negative pressure head setting $h_b = -0.40$ m and using the λ and η parameters generated by VS2DT shown in Tables 4 and 6 for t = 10,560 min.

λ and h_b), it is worthwhile to compare measured soil moisture at the various neutron access tubes with simulated soil moisture from VS2DT output using the parameters in Table 6. Such a comparison is made in Figure 10 for the three times 480, 2220, and 10,560 min and at three locations MBN-5, MBN-10, and MBN-15. The best agreement between measured and simulated moisture contents occurs in the range of moisture contents greater than 0.20 for all three times, probably because it has the greatest influence on drawdown in the saturated zone. Also, upon close inspection of Figure 10, it can be seen that the best agreement occurs in access tube MBN-15 located at the largest distance from the pumped well. The closer that an access tube is to the pumped well, the greater the elongation of the capillary fringe (Table 5) and the greater the model discrepancy. The model does not properly simulate low (<0.20) moisture contents because of the likelihood of numerical errors in VS2DT and to the exceedingly low RHC predicted by the Assouline model with large values of η.

4. Summary and Discussion

[54] Saturated zone parameters estimated with the Moench model are shown in Table 2 and are deemed accurate on the basis of what is known about the aquifer composition and dimensions. Figure 3 shows that the agreement between measured and simulated drawdowns is, with the exception of early and intermediate-time drawdowns in piezometers P2 and P18 and to a lesser degree in piezometers P1 and WD1A, quite good suggesting that the aquifer is fairly homogeneous at the scale of the test. The Moench model uses a set of three adjustable empirical fitting parameters in the water table boundary condition to account for effects of drainage from the zone above the water table. Because of the number of quasi-independent fitting parameters and low root-mean-square error (0.0228 m), the

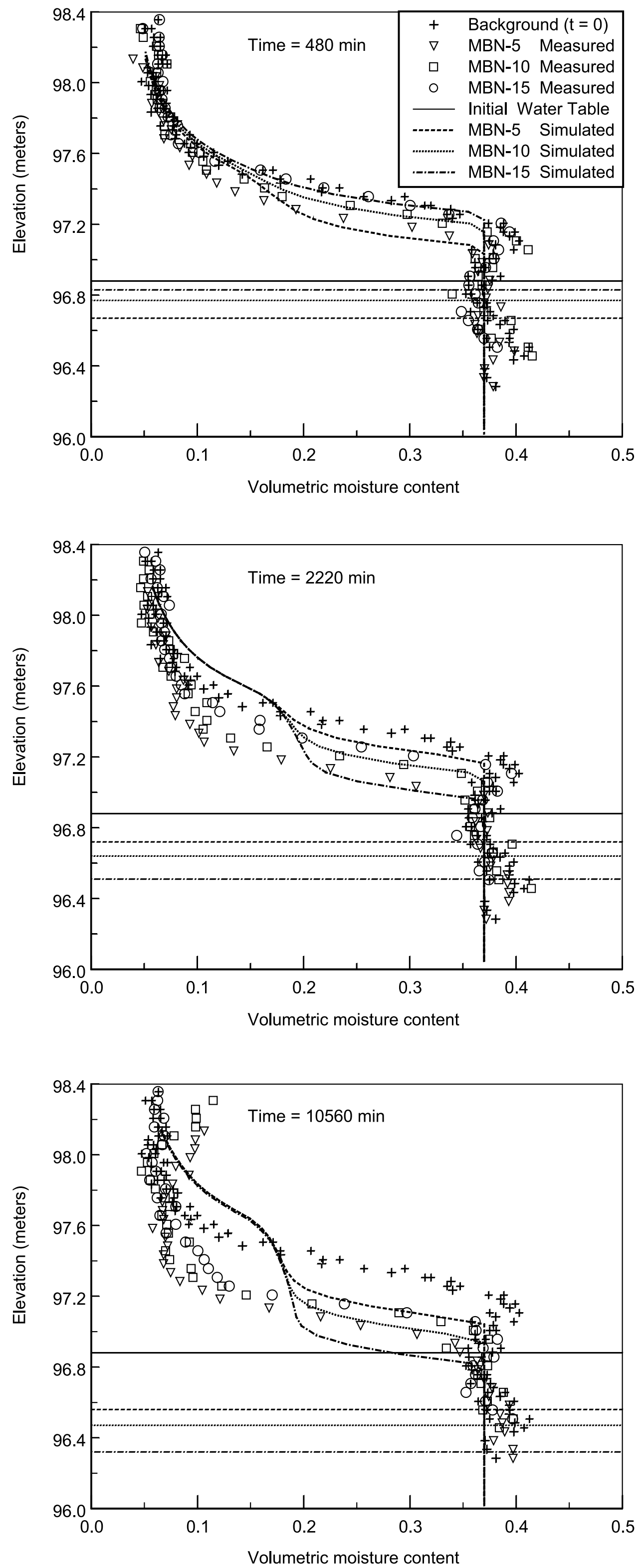

Figure 10. Measured and simulated volumetric moisture contents at MBN5, MBN10, and MBN15 at the specified times and the corresponding simulated locations of the water table. Moisture distributions and locations of the water table are simulated using parameters in Table 6.

Moench model is assumed to yield accurate values of saturated zone hydraulic parameters against which parameters estimated with other models used in this paper can be compared.

[55] In order to bring unsaturated zone hydraulic characteristics into consideration, the model proposed by *Mathias and Butler* [2006] was used. In this model the soil moisture distribution in the unsaturated zone (above the capillary fringe) is represented by an exponential equation with the moisture retention exponent a_c, and the RHC is represented by an exponential equation with the relative permeability exponent a_k. Comparisons (not shown) of measured drawdowns with simulated drawdowns using the estimated parameters in Table 3a where $a_c \neq a_k$ or in Table 3b where $a_c = a_k$ appear nearly identical with one another and with the simulations in Figure 3. The goodness of fit can be assessed roughly by referring to the values of RMSE in Tables 2, 3a, and 3b. Results obtained with $a_c = a_k$ yield a completely unrealistic soil moisture retention ($a_c = 0.228$ m^{-1} in equation (1)) as it bears no resemblance to measured soil moisture (Figure 4a. Results obtained with $a_c \neq a_k$ with $a_c = 5.0$ m^{-1}, which was estimated by fitting equation (1) to soil moisture measurements, yield a RHC relation ($a_k = 31.7$ m^{-1} in equation (2)) that declines steeply with negative capillary pressure head above the top of the capillary fringe (Figure 4b). Forcing $a_c = a_k = 5.0$ m^{-1} was found to degrade the comparison between measured and simulated drawdowns. The implication here is that models that require $a_c = a_k$ such as those of *Kroszynski and Dagan* [1975] and *Tartakovsky and Neuman* [2007] are inappropriate for the Borden site aquifer test and perhaps for other aquifer tests as well. *Tartakovsky and Neuman* [2007, p. 3] have stated that their model can "...be extended to include two separate exponents, finite unsaturated zone thickness and borehole storage." Because their model allows for both horizontal and vertical flow in the unsaturated zone, the publication of a paper extending the model in this manner will be a welcome addition to the aquifer test literature.

[56] Flow vectors seen with the graphical user interface for VS2DT [*Hsieh et al.*, 2000] show the flow in the region above the capillary fringe to be more horizontal than vertical and thereby provide support for the *Tartakovsky and Neuman* [2007] model. However, comparison of Figure 7 wherein horizontal flow in the unsaturated zone is included in the model with Figure 4c wherein horizontal flow in the unsaturated zone is not included, shows that little benefit accrues from components of horizontal flow being included in the mathematical development and much is lost by requiring $a_c = a_k$ in the analytical models. As a reminder, the simulated early time drawdowns exceed the measured drawdowns in Figure 7 because delayed piezometer response is not included in the numerical model.

[57] Use of the *Brooks and Corey* [1964] functional relations in the numerical model VS2DT to describe the hydraulic characteristics at the Borden site resulted in saturated zone vertical and horizontal hydraulic conductivities that agree well with those obtained analytically. The set of hydraulic parameters in Table 4 yields simulated drawdowns in excellent agreement with measured drawdowns at all but very early time; however, the soil moisture retention does not resemble measured volumetric soil moisture profiles (Figure 6). The numerical model in this instance, like the analytical model, provides only a means to fit simulated drawdowns to measured drawdowns. The soil moisture retention is similar to that obtained by *Moench* [2003] who used the Brooks and Corey formulation and VS2DT, without the benefit of soil moisture measurements, to obtain soil moisture characteristics for the Cape Cod, MA site. The resulting soil moisture retention curve for the Cape Cod site, like that in Figure 6, has a shape similar to that for fine-grained material rather than for the well-sorted, coarse-grained material of which the aquifer is known to be composed.

[58] The Assouline model overcomes the inability of the estimated pore size distribution index λ in the Brooks and Corey formulation to yield a match between soil moisture retention and measured soil moisture profiles and at the same time provide satisfactory agreement between simulated and measured drawdowns in piezometers. The Assouline model for the RHC retains the Brooks and Corey soil moisture relation and introduces the parameter η (equation (5)) that relates to soil structure and texture. This revision provides the necessary flexibility in the model but adds another parameter to estimate.

[59] Modification of VS2DT by replacing the Brooks and Corey RHC with the Assouline RHC and minimizing the squared differences between measured and simulated drawdowns result in the exponent in equation (5), namely $-\eta - \eta\lambda$, being significantly larger than the exponent in equation (4), namely $-2-3\lambda$. The finding indicates that the RHC (see Figure 9) declines more rapidly with increased negative pressure head than it does with the standard Brooks Corey relation. The values of η in Table 6 are considerably larger than values obtained for core samples by *Assouline* [2001] and appear to be a consequence of field-scale soil structure and texture (see the quotation from *Sudicky* [1986] in section 2).

[60] *Brooks and Corey* [1964] developed their functional relation for relative hydraulic conductivity by using the capillary bundle model of *Burdine* [1953], which is based on concepts of hydraulic radius and tortuosity as applied to flow in unsaturated porous media. From the analysis presented here for the Borden site aquifer test, it would appear that this model does not apply at the field scale. The Assouline model, which uses the approach of *Mualem* [1976] and introduces the parameter η, appears to be an improvement. One might ask why this is so. First to consider is the inherent heterogeneity in aquifer materials at the scale of the aquifer test as documented for the Borden site [e.g., *Sudicky*, 1986], which doubtless extends to the zone above the water table. A second consideration is the presence of flow in the capillary fringe and in the unsaturated zone above it, which is oriented more in the horizontal direction toward the pumping well than in the vertical direction toward the water table. As the aquifer at the Borden site is anisotropic, flow paths in the unsaturated zone parallel to particle orientation in the glaciodeltaic or glaciofluvial deposits would likely involve a different tortuosity and hydraulic radius than flow paths perpendicular to particle orientation.

[61] From the VS2DT computer output, simulated soil moisture values were compared with measured values obtained in three neutron access tubes (MBN-5, MBN-10, and MBN-15). Simulated soil moistures in Figure 10 show reasonably accurate representations of the actual soil mois-

ture in the moist range with best agreement at the more distant neutron access tubes and poorer agreement closer to the pumped well, which is attributed to failure of VS2DT to account for variation in the hydraulic properties in the radial direction. That the model simulations in the moist range agree quite well with measured profiles at 480 and 2220 min at all three locations in Figure 10 is attributed to the fact that soil moisture redistribution in response to a falling water table occurs rapidly because of relatively high hydraulic head gradients in the early part of the aquifer test. Considering that the simulations plotted in Figure 10 were based only on drawdown measurements in the saturated zone, knowledge of the height of the capillary fringe at each of the indicated times and the shape of the initial soil moisture profile, it would appear that the model performs reasonably well in the range of moisture contents above 0.20. A possible explanation for the poor performance at low moisture contents is the exceedingly low RHC in the drier range.

[62] Regarding possible mechanisms for the observed capillary fringe elongations, it is of interest to note that similar effects have been observed in unsteady state or "dynamic" laboratory column experiments when compared with steady state or equilibrium methods [e.g., *Topp et al.*, 1967; *Vachaud et al.*, 1972; *Elzeftawy and Mansell*, 1975; *Stauffer*, 1978]. In the soils literature such phenomena are referred to as the flow rate dependence of soil hydraulic characteristics. More recently they have been observed in experiments by *Wildenschild et al.* [2001] and *O'Carroll et al.* [2005]. Many mechanisms have been proposed for these reported observations [see *Wildenschild et al.*, 2001; *Hassanizadeh et al.*, 2002; *O'Carroll et al.*, 2005] but none confirmed or considered definitive. Whatever the proposed mechanism, it would require accounting for (1) elongation of the capillary fringe, which continues unabated to the end of the pumping test, and (2) flow rate dependency of the capillary fringe elongations, which is inferred from the correspondence between the radial variation in h_b and the radial variation in elevation of the water table. One mechanism, frequently invoked in the soils literature, that seems to satisfy these requirements for the aquifer test is the entrapment of water in pores (or groups of pores) isolated during the drainage process [see, e.g., *Topp et al.*, 1967; *Wildenschild et al.*, 2001] or a lack of air-phase continuity such as observed by *Silliman et al.* [2002] while conducting 2-D sand tank experiments with overlapping layers of fine- and coarse-grained materials. Related to these mechanisms is the possibility that the drainage and imbibition history due to seasonal fluctuations of the water table may bring on capillary fringe elongation in the course of an aquifer test.

[63] An alternative mechanism that would appear to satisfy the above two requirements is a reduction in porosity due to increased tension in response to the declining water table. Along these lines, *Vachaud et al.* [1972, p. 532] noted that the observed behavior "...is similar to the stress deformation relationship of viscoelastic material, where the deformation obtained for a given stress is dependent on the rate of the stress." This mechanism brings into question the applicability to the Borden site sediments of the rigid porous medium assumption upon which the numerical model VS2DT is based. Although the imposed added tension due to water table drawdown is small, the mechanism is suggested here because of the poorly consolidated nature of the aquifer materials at the Borden site. Porosity reduction in the zone above the water table would simultaneously raise the height of the capillary fringe and reduce the saturated hydraulic conductivity within the capillary fringe. Also, such soil deformation might account for the capillary fringe elongation being sustained throughout the length of the 7-d aquifer test. Relevant here but not applied are models suggested by *Assouline* [2006a, 2006b] to account for the influence of changes in bulk density upon the soil moisture retention and RHC functions.

5. Conclusions

[64] The analyses presented in this paper suggest that it is indeed possible to make large-scale estimates of soil moisture characteristics from a detailed unconfined aquifer test such as that conducted by *Bevan* [2002] in August 2001 at the Borden site. For the purpose of estimating saturated zone aquifer parameters only, the Moench model provides a "best fit" standard for comparison with other models used in this paper. Analytical and numerical analyses conducted with different models designed for the purpose of estimating both saturated and unsaturated zone hydraulic parameters show that the relative hydraulic conductivity (RHC) function must contain a fitting parameter that is different from the fitting parameter used in the soil moisture distribution function. This requirement is satisfied with the analytical model of *Mathias and Butler* [2006] using exponential functional relations and with the USGS numerical model VS2DT combined with a model proposed by the *Assouline* [2001]. The latter uses functional relations similar to those of *Brooks and Corey* [1964] but with a simple modification that introduces a parameter η into the RHC function that relates to soil structure and texture. From the aquifer test analysis the η parameter is estimated to be many times greater than values determined from core samples [*Assouline*, 2001] and may relate to the presence in the aquifer of discontinuous lenses of fine-, medium-, and coarse-grained sand. It is concluded that field-scale RHC declines more rapidly with elevation above the top of the capillary fringe than would be expected if the parameters were to be based on core-scale measurements and analyses.

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References

Akindunni, F. F., and R. W. Gillham (1992), Unsaturated and saturated flow in response to pumping of an unconfined aquifer: Numerical investigation of delayed drainage, *Ground Water*, *30*, 873–884, doi:10.1111/j.1745-6584.1992.tb01570.x.

Assouline, S. (2001), A model for soil relative hydraulic conductivity based on the water retention characteristic curve, *Water Resour. Res.*, *37*(2), 265–271, doi:10.1029/2000WR900254. (Correction, *Water Resour. Res.*, *40*(2), W02901, doi:10.1029/2004WR003025, 2004.)

Assouline, S. (2006a), Modeling the relationship between soil bulk density and the water retention curve, *Vadose Zone J.*, *5*, 554–563, doi:10.2136/vzj2005.0083.
Assouline, S. (2006b), Modeling the relationship between soil bulk density and the hydraulic conductivity function, *Vadose Zone J.*, *5*, 697–705, doi:10.2136/vzj2005.0084.
Barlow, P. M., and A. F. Moench (1999), WTAQ-A computer program for calculating drawdowns and estimating hydraulic properties for confined and water-table aquifers, U.S. Geol. Surv. Water Resour. Invest. Rep., 99-4225. (Available at http://ma.water.usgs.gov/publications/WRIR_99-4225/index.htm)
Berkowitz, B., S. E. Silliman, and A. M. Dunn (2004), Impact of the capillary fringe on local flow, chemical migration, and microbiology, *Vadose Zone J.*, *3*, 534–548.
Bevan, M. J. (2002), A detailed study of water content variation during pumping and recovery in an unconfined aquifer, M.Sci. thesis, Univ. of Waterloo, Waterloo, Ont., Canada.
Bevan, M. J., A. L. Endres, D. L. Rudolph, and G. Parkin (2005), A field scale study of pumping-induced drainage and recovery in an unconfined aquifer, *J. Hydrol.*, *315*, 52–70, doi:10.1016/j.jhydrol.2005.04.006.
Boulton, N. S. (1954), Unsteady radial flow to a pumped well allowing for delayed yield from storage, *Publ. 37*, pp. 472–477, Int. Assoc. of Sci. Hydrol., Rome.
Boulton, N. S. (1963), Analysis of data from non-equilibrium pumping tests allowing for delayed yield from storage, *Proc. Inst. Civ. Eng.*, *26*, 469–482.
Brooks, R. H., and A. T. Corey (1964), Hydraulic properties of porous media, *Hydrol. Pap. 3*, 27 pp., Colo. State Univ., Fort Collins, Colo.
Burdine, N. T. (1953), Relative permeability calculations from pore size distribution data, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *198*, 71–78.
Dagan, G. (1967), A method of determining the permeability and effective porosity of unconfined anisotropic aquifers, *Water Resour. Res.*, *3*, 1059–1071, doi:10.1029/WR003i004p01059.
de Hoog, F. R., J. H. Knight, and A. N. Stokes (1982), An improved method for numerical inversion of Laplace transforms, *SIAM J. Sci. Comput.*, *3*, 357–366, doi:10.1137/0903022.
El-Kadi, A. I. (2005), Validity of the generalized Richards equation for the analysis of pumping test data for a coarse-material aquifer, *Vadose Zone J.*, *4*, 196–205.
Elzeftawy, A., and R. S. Mansell (1975), Hydraulic conductivity calculations for unsaturated steady-state and transient-state flow in sand, *Soil Sci. Soc. Am. J.*, *39*, 599–603.
Endres, A. L., J. P. Jones, and E. A. Bertrand (2007), Pumping-induced vadose zone drainage and storage in an unconfined aquifer: A comparison of analytical model predictions and field measurements, *J. Hydrol.*, *335*, 207–218, doi:10.1016/j.jhydrol.2006.07.018.
Gardner, W. R. (1958), Some steady state solutions of the unsaturated moisture flow equations with application to evaporation from a water table, *Soil Sci.*, *85*, 228–232.
Ghezzehei, T. A., T. J. Kneafsey, and G. W. Su (2007), Correspondence of the Gardner and van Genuchten-Mualem relative permeability function parameters, *Water Resour. Res.*, *43*, W10417, doi:10.1029/2006WR005339.
Hassanizadeh, S. M., M. A. Celia, and H. K. Dahle (2002), Dynamic effect in the capillary pressure-saturation relationship and its impacts on unsaturated flow, *Vadose Zone J.*, *1*, 38–57.
Healy, R. W. (1990), Simulation of solute transport in variably saturated porous media with supplemental information on modifications to the U.S. Geological Survey's computer program VS2D, *U.S. Geol. Surv. Water Resour. Invest. Rep., 90-4025*, 125 pp. (Available at http://pubs.er.usgs.gov/pubs/wri/wri904025)
Heywood, C. (1995), Investigation of aquifer-system compaction in the Hueco Basin, El Paso, Texas, USA, in *Land Subsidence: Proceedings of the Fifth International Symposium on Land Subsidence, the Hague*, edited by F. B. J. Barends, F. J. J. Brouwer, and F. H. Schröder, *IAHS Publ., 234*, 35–45.
Hsieh, P. A., W. Wingle, and R. W. Healy (2000), VS2DI-A graphical software package for simulating fluid flow and solute or energy transport in variably saturated porous media, *U.S. Geol. Surv. Water Resour. Invest. Rep., 99-4130*, 16 pp. (Available at http://water.usgs.gov/software/ground_water.html)
Kroszynski, U. I., and G. Dagan (1975), Well pumping in unconfined aquifers: The influence of the unsaturated zone, *Water Resour. Res.*, *11*, 479–490, doi:10.1029/WR011i003p00479.
Kueper, B. H., and E. O. Frind (1991), Two-phase flow in heterogeneous porous media: 2. Model application, *Water Resour. Res.*, *27*, 1059–1070, doi:10.1029/91WR00267.
Lappala, E. G., R. W. Healy, and E. P. Weeks (1987), Documentation of computer program VS2D to solve the equations of fluid flow in variably saturated porous media, *U.S. Geol. Surv. Water Resour. Invest. Rep., 83-4099*, 184 pp. (Available at http://pubs.er.usgs.gov/pubs/wri/wri834099)
MacFarlane, D. S., J. A. Cherry, R. W. Gillham, and E. A. Sudicky (1983), Migration of contaminants in groundwater at a landfill: A case study 1. Groundwater flow and plume delineation, *J. Hydrol.*, *63*, 1–29, doi:10.1016/0022-1694(83)90221-4.
Mathias, S. A., and A. P. Butler (2006), Linearized Richards' equation approach to pumping test analysis in compressible aquifers, *Water Resour. Res.*, *42*, W06408, doi:10.1029/2005WR004680.
Moench, A. F. (1997), Flow to a well of finite diameter in a homogeneous, anisotropic water table aquifer, *Water Resour. Res.*, *33*, 1397–1407, doi:10.1029/97WR00651.
Moench, A. F. (2003), Estimation of hectare-scale soil-moisture characteristics from aquifer-test data, *J. Hydrol.*, *281*, 82–95, doi:10.1016/S0022-1694(03)00202-6.
Moench, A. F. (2004), Importance of the vadose zone in analyses of unconfined aquifer tests, *Ground Water*, *42*, 223–233, doi:10.1111/j.1745-6584.2004.tb02669.x.
Moench, A. F., S. P. Garabedian, and D. R. LeBlanc (2001), Estimation of hydraulic parameters from an unconfined aquifer test conducted in a glacial outwash deposit, Cape Cod, Massachusetts, *U.S. Geol. Surv. Prof. Pap., 1629*, 69 pp. (Available at http://water.usgs.gov/pubs/pp/pp1629/pdf/pp1629ver2.pdf)
Mualem, Y. (1976), A new model for predicting the hydraulic conductivity of unsaturated porous media, *Water Resour. Res.*, *12*, 513–522, doi:10.1029/WR012i003p00513.
Narasimhan, T. N., and M. Zhu (1993), Transient flow of water to a well in an unconfined aquifer: Applicability of some conceptual models, *Water Resour. Res.*, *29*, 179–191, doi:10.1029/92WR01959.
Neuman, S. P. (1972), Theory of flow in unconfined aquifers considering delayed response of the water table, *Water Resour. Res.*, *8*, 1031–1044, doi:10.1029/WR008i004p01031.
Neuman, S. P. (1974), Effects of partial penetration on flow in unconfined aquifers considering delayed aquifer response, *Water Resour. Res.*, *10*, 303–312, doi:10.1029/WR010i002p00303.
Nwankwor, G. I. (1985), Delayed yield processes and specific yield in a shallow sand aquifer, Ph.D. thesis, Univ. of Waterloo, Waterloo, Ont., Canada.
Nwankwor, G. I., J. A. Cherry, and R. W. Gillham (1984), A comparative study of specific yield determinations for a shallow sand aquifer, *Ground Water*, *22*, 764–772, doi:10.1111/j.1745-6584.1984.tb01445.x.
Nwankwor, G. I., R. W. Gillham, G. van der Kamp, and F. F. Akindunni (1992), Unsaturated and saturated flow in response to pumping of an unconfined aquifer-Field evidence of delayed drainage, *Ground Water*, *30*, 690–700, doi:10.1111/j.1745-6584.1992.tb01555.x.
O'Carroll, D. M., T. J. Phelan, and L. M. Abriola (2005), Exploring dynamic effects in capillary pressure in multistep outflow experiments, *Water Resour. Res.*, *41*, W11419, doi:10.1029/2005WR004010.
Silliman, S. E., B. J. Berkowitz, J. Simunek, and M. T. van Genuchten (2002), Fluid flow and solute migration within the capillary fringe, *Ground Water*, *40*, 76–84, doi:10.1111/j.1745-6584.2002.tb02493.x.
Stauffer, F. (1978), Time dependence of the relations between capillary pressure, water content and conductivity during drainage of porous media, paper presented at Symposium on Scale Effects in Porous Media, Int. Assoc. for Hydraul. Res., Thessaloniki, Greece, 29 Aug. to 1 Sept.
Stehfest, H. (1970), Numerical inversion of Laplace transforms, *Commun. ACM*, *13*(1), 47–49, doi:10.1145/361953.361969.
Sudicky, E. A. (1986), A natural gradient experiment on solute transport in a sand aquifer: Spatial variability of hydraulic conductivity and its role in the dispersive process, *Water Resour. Res.*, *22*, 2069–2082, doi:10.1029/WR022i013p02069.
Tartakovsky, G. D., and S. P. Neuman (2007), Three-dimensional saturated-unsaturated flow with axial symmetry to a partially penetrating well in a compressible unconfined aquifer, *Water Resour. Res.*, *43*, W01410, doi:10.1029/2006WR005153.
Topp, G. C., A. Klute, and D. B. Peters (1967), Comparison of water content-pressure head data obtained by equilibrium, steady-state, and unsteady-state methods, *Soil Sci. Soc. Am. Proc.*, *31*, 312–314.
Vachaud, G., M. Vauclin, and M. Wakil (1972), A study of the uniqueness of the soil moisture characteristic during desorption by vertical drainage, *Soil Sci. Soc. Am. Proc.*, *36*, 531–532.
van Genuchten, M. T. (1980), A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am. J.*, *44*, 892–898.
Wildenschild, D., J. W. Hopmans, and J. Simunek (2001), Flow rate dependence of soil hydraulic characteristics, *Soil Sci. Soc. Am. J.*, *65*, 35–48.

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